University of California, Berkeley **Economics 230a**
Department of Economics **Fall 2013**

Problem Set #2

(due 11/26/13)

1. Consider an economy in which there are two commodities, musical composition (X) and home construction (Y). Each is produced competitively using a single factor, labor, which is in fixed supply. Producers of X face the production function $X = (\alpha e^{-\beta Y})L_x$, and producers of Y face the production function $Y = \gamma L_y$, where $L = L_x + L_y$ is total labor supply. That is, each producer perceives constant returns to scale with respect to labor input, but the productivity of labor in sector X is reduced via a negative production externality by the aggregate level of production of Y.

   A. Derive the economy's production possibilities frontier as an expression for X in terms of Y, and show that the production set is not convex.

   B. Letting labor be numeraire, derive the cost functions for producers of each good, $c(X;Y)$ and $c(Y)$, assuming that producers of X take Y as given and that producers of Y ignore their impact on sector X. Solve for the competitive prices at given values of X and Y.

   C. Now, solve for the social cost function for X and Y, $c(X,Y)$, and the marginal social costs of X and Y at given production levels. Derive expressions for Pigouvian taxes on producers of X and Y that would cause competitive prices to equal marginal social costs.

   D. Suppose that consumers are identical, with preferences that satisfy the utility function $U(X,Y) = X+Y$. How much revenue does the Pigouvian tax raise at the social optimum?

2. Suppose that an economy has two goods, education and housing, and that every family has preferences over the two goods defined by the common utility function, $U(E,H) = E^{\alpha}H^{1-\alpha}$. Households differ only with respect to income level, with household i's exogenous income equal to y^i. Housing is produced subject to constant unit cost $p_H = 1$, and may be purchased in any quantity. Education may be produced using one of two technologies: by the private sector as a regular private good with unit cost $p_E = 1$ per family, and by the public sector as a *pure public good* with unit cost q per unit of the common level of public education. Publicly provided education is financed by a proportional tax at rate τ on income, and no individual household may use public and private education at the same time.

   A. For fixed values of the tax rate, τ, and the level of public education, G, show that there exists a critical level of income, $\hat{y}$, above which households choose private school, and below which households choose public school. Show that $\hat{y}$ is increasing in G, given τ.

   B. Start with your solution for $\hat{y}$ as a function of G and τ from part A. Letting Y equal aggregate income in the economy, substitute for τ using the government's budget constraint that relates τ to G and Y. Calculate the full effect of G on $\hat{y}$, i.e., the effect taking into account the impact of G on τ. Show that this effect is larger than the partial effect you solved for in part A, and explain why.

C. Show that, among individuals who choose public education, there single level of public education, say G^*, that is most preferred by all. If the existing level of public education is initially at G^*, under what condition would a majority of the overall population vote for a small decrease in spending on public education spending? (*Hint*: relate $\hat{y}$ at G^* to the income of the median voter.)

3. Consider an individual who wishes to invest initial wealth, W, to maximize the utility of terminal wealth one period hence. The investor's problem consists of two decisions:

   (1) how much of this wealth to place in bonds, which yield a certain return, $i > 0$, and how much to invest in stocks, which yield a stochastic return $r \in [0, R]$, $E(r) = \bar{r} > i$;

   (2) how to allocate these assets between a taxable account and a tax-sheltered account.

   Interest on bonds held in the taxable account (*TA*) is taxed at rate τ $(0 < \tau < 1)$, while equity returns are taxed at rate $\lambda\tau$ $(0 < \lambda < 1)$. Assets placed in the tax-sheltered account (*TSA*) are tax-exempt. An amount up to $V < W$ may be placed in the tax-sheltered account.

   A. Derive the optimal portfolio, in terms of the amounts of debt and equity held in each account, for an individual who is risk neutral; perform the same exercise for an individual who is infinitely risk averse.

   B. Show that, regardless of an individual's degree of risk aversion, it will never be optimal for the individual to hold equity in the *TSA* and bonds in the *TA* at the same time. (Hint: starting with such an initial allocation, show that a portfolio shift would permit the investor to achieve higher aggregate after-tax earnings on debt for a given distribution of aggregate after-tax earnings on equity.)